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Stochastic Filtering

In many areas of human endeavour, the systems involved are not available for direct measurement. Instead, by combining mathematical models for a systems evolution with partial observations of its evolving state, we can make reasonable inferences about it.

The celebrated Kalman-Bucy filter, designed for linear dynamical systems with linearly structured measurements, is the most famous Bayesian filter. Its generalizations to nonlinear systems and/or observations are collectively referred to as stochastic (or nonlinear) filtering, an extension of the Bayesian framework to the estimation, prediction, and interpolation of nonlinear stochastic dynamics. Stochastic filtering uses a stochastic model to make inferences about an evolving system and is a theoretically optimal algorithm.

The breadth of its applications, firmly established and still emerging, is simply astounding. Early uses include cryptography, tracking, and guidance. Nowadays, the scope of stochastic filtering includes the study of global climate, estimating the state of the economy, identifying tumours using non-invasive methods, and much more.

This course is intended to provide an introduction to the problem of stochastic filtering and the numerical methods used to solve it.

Syllabus:

- The Filtering Framework
- The Filtering Equations: The change of probability measure approach
- The Innovation Process
- Uniqueness of the solutions to the Zakai and Kushner-Stratonovitch Equations.
- Finite Dimesional Filters. The Kalman Bucy Filter. The Benes Filter.
- Numerical methods for solving the filtering problem.
- Particle filters (Sequential Monte Carlo Methods)
- An application of the Kusuoka-Lyons-Victoir method to filtering.