Theory of Capacity, G-expectation and application to pricing and hedging under uncertainty

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Abstract :
In this lecture, we shall present new advances in pricing and hedging under uncertainty based on capacity theory, the new tool of G-expectation and backward stochastic differential equation (BSDE's).
To price and hedge under model uncertainty (such as in the *Uncertain Volatility Model* in [1, 5]), typically we deal with a family of martingale measures which is non-dominated, so that we can not work under an ad-hoc risk neutral probability measure. Therefore, we have to develop a *quasi-sure* stochastic calculus to give a sense to the objects that we consider under this non-dominated set of probability measures.

Plan of the course :

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Course 1	Motivation and introduction to pricing and hedging under uncertainty	Introduction to BSDE's and g-expectation
Course 2	Nonlinear expectation and representation theorem	Link with nonlinear PDE's in [1] and [5]
Course 3	Choquet Capacity and Application to G-expectation	G-expectation
Course 4	Quasi-sure stochastic analysis	Second order BSDE's
Course 5	Applications to finance	Applications to finance

References

[1] Avellaneda, M., Levy, A. and Paras, A. : Pricing and hedging derivatives securities in markets with uncertain volatilities. Appl. Math. Finance 2, 73-88 (1995).

[2] Choquet, G. "Theory of capacities" *Annales de l'Institut Fourier* 5, pp 131-295 (1955).

[3] Denis, L. and Martini, C. "A theoretical framework for the pricing of contingent claims in the presence of model uncertainty", *The Ann. of Appl. Probability* 16(2), 827-852 (2006).

[4] Denis, L., Hu, M. and Peng, S. " Function spaces and capacity related to a Sublinear Expectation: application to G-Brownian Motion Pathes", to appear in *Potential Analysis*, available on *ArKiv*

[5] Lyons, T.J. "Uncertain volatility and the risk-free synthesis of derivatives", *Journal of Applied Finance* Vol 2 (1995)

[6] Peng, S. "Nonlinear Expectation and Stochastic Calculus under Uncertainty" *arXiv, Feb. 24 (2010).*

[7] Soner, M., Touzi, N. and Zhang, Jianfeng "Martingale Representation Theorem for the G-Expectation", *arXiv*:1001.3802v2..

[8] Soner, M., Touzi, N. and Zhang, Jianfeng "Wellposedness of second order backward SDEs", *arXiv*:1003.6053v2..