Malliavin calculus for Lvy processes and applications to finance

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Abstract

The Malliavin calculus was originally introduced by Paul Malliavin in 1978 as a tool to study smoothness of densities of solutions of stochastic differential equations. This was a rather restricted scope of applications and the theory was difficult, so for more than 10 years this was a topic of interest to only a limited group of experts.

However, when Ocone in 1994 showed that Malliavin calculus could be used to obtain an explicit version of the It representation theorem (now known as the Clark-Ocone formula), and when subsequently Karatzas and Ocone applied this to finance, the interest in this area exploded. At the same time simpler presentations of the theory were developed. Soon even bank employees started studying Malliavin calculus!

The Malliavin calculus was first introduced for Brownian motion, but it has later been extended to Lévy processes. At the same time new applications of the theory have been discovered. In this course we give a simple introduction to Malliavin calculus for Lvy processes and we give examples of applications to finance. Here is an outline of the course:

Lecture 1: Introduction to Malliavin calculus for Brownian motion (i): The Malliavin derivative, or Hida-Malliavin derivative, as a stochastic gradient

Lecture 2: Malliavin calculus for Brownian motion (ii): The Malliavin derivative by means of chaos expansion. Properties of the Malliavin derivative, including the chain rule, the duality theorem and the fundamental theorem of stochastic calculus.

Lecture 3: Malliavin calculus for Brownian motion (iii): Applications: (a)The Clark-Ocone formula and applications to hedging (b) Sensitivity results and application to efficient numerical computation of the "greeks" in finance.

Lecture 4: Malliavin calculus for Lvy processes (i). Introduction to stochastic calculus for Lvy processes. The Malliavin derivative by means of chaos expansion. Properties of the Malliavin derivative. The Clark-Ocone formula revisited. Hedging in incomplete markets.

Lecture 5: Malliavin calculus for Lévy processes (ii). Applications, for example the following:

(a) Minimal variance portfolio in incomplete markets

(b) Optimal portfolio with partial information

The presentation is based on the 3 first chapters of Part 1 and Part 2, respectively, of the book by Giulia Di Nunno, Bernt Oksendal and Frank Proske, entitled "Malliavin Calculus for Lévy Processes and Applications to Finance", Universitext, Springer 2009.